What is the golden section (or Phi)?

We will call the Golden Ratio (or Golden number) after a greek letter, **Phi** (φ) here, although some writers and mathematicians use another Greek letter, **tau** (τ).

A bit of history...

Euclid, the Greek mathematician of about 300BC, wrote the *Elements* which is a collection of 13 books on Geometry (written in Greek originally). It was the most important mathematical work until this century, when Geometry began to take a lower place on school syllabuses, but it has had a major influence on mathematics.

It seems that this ratio had been of interest to earlier Greek mathematicians, especially Pythagoras (580BC - 500BC) and his "school". There is an interesting article on The Golden ratio at the St Andrew's MacTutor History of Mathematics site.

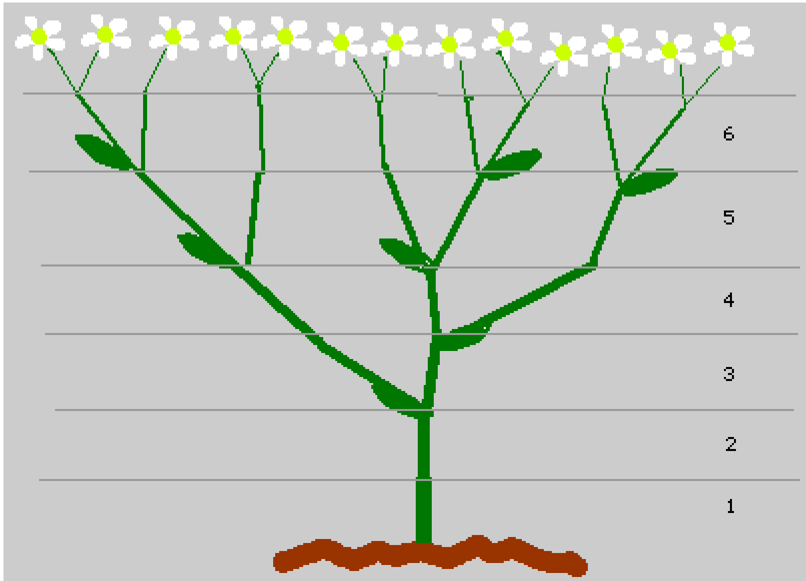
Phi has the value $(\sqrt{5} + 1)/2$

If we take the ratio of two successive numbers in Fibonacci's series, (1, 1, 2, 3, 5, 8, 13, ..) and we divide each by the number before it, we will find the following series of numbers:

${}^{1}/_{1} = 1$, ${}^{2}/_{1} = 2$, ${}^{3}/_{2} = 1{\cdot}5$, ${}^{5}/_{3} = 1{\cdot}666...$, ${}^{8}/_{5} = 1{\cdot}6$, ${}^{13}/_{8} = 1{\cdot}625$, ${}^{21}/_{13} = 1{\cdot}61538...$

The ratio seems to be settling down to a particular value, which we call **the golden ratio** or **the golden number**. It has a value of approximately **1·618034.**

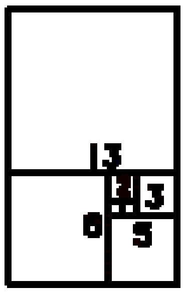
Fibonacci Numbers, the Golden Section and Plants



One plant in particular shows the Fibonacci numbers in the number of "growing points" that it has. Suppose that when a plant puts out a new shoot, that shoot has to grow two months before it is strong enough to support branching. If it branches every month after that at the growing point, we get the picture shown here.

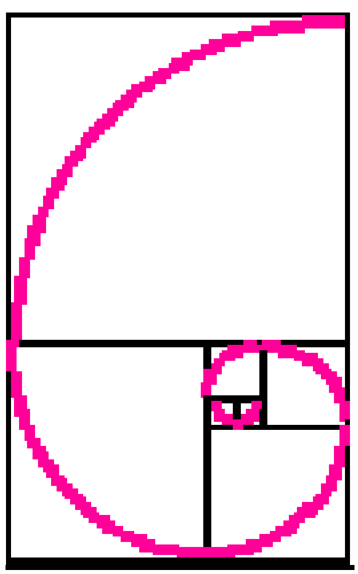
A plant that grows very much like this is the "sneezewort": Achillea ptarmica.

Fibonacci Rectangles and Shell Spirals



We can make another picture showing the Fibonacci numbers 1,1,2,3,5,8,13,21,.. if we start with two small squares of size 1 next to each other. On top of both of these draw a square of size 2 (=1+1).

We can now draw a new square - touching both a unit square and the latest square of side 2 - so having sides 3 units long; and then another touching both the 2-square and the 3-square (which has sides of 5 units). We can continue adding squares around the picture, **each new square having a side which is as long as the sum of the latest two square's sides**. This set of rectangles whose sides are two successive Fibonacci numbers in length and which are composed of squares with sides which are Fibonacci numbers, we will call the **Fibonacci Rectangles**.



Here is a spiral drawn in the squares, a quarter of a circle in each square. The spiral is not a *true* mathematical spiral (since it is made up of fragments which are parts of circles and does not go on getting smaller and smaller) but it is a good approximation to a kind of spiral that does appear often in nature. Such spirals are seen in the shape of shells of snails and sea shells and, as we see later, in the arrangement of seeds on flowering plants too. The spiral-in-the-squares makes a line from the centre of the spiral increase by a factor of the golden number in each square. So points on the spiral are 1.618 times as far from the centre after a quarter-turn. In a whole turn the points on a radius out from the center are $1.618^4 = 6.854$ times further out than when the curve last crossed the same radial line.